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Hawking Effect in Vaidya–Bonner Space-Time

Zhu Jianyang,[1,2] Zhang Jianhua,[1,3] and Zhao Zheng[1]

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A new method determining the location and the temperature of event horizons of evaporating black holes is suggested. Both the Klein–Gordon equation and the Dirac equation are studied with the method in a Vaidya–Bonner space-time. These equations are reduced near the event horizon when the generalized tortoise coordinates are adopted. The location and the temperature of the event horizon are shown automatically. The first approximation of our result is the same as that obtained by using the calculation of the vacuum expectation value of the renormalized energy-momentum tensor when the evaporation of the black hole is very slow.

It is difficult to determine the location and the temperature of the event horizon of an evaporating black hole because the calculation of the vacuum expectation value of the renormalized energy-momentum tensor is very complicated (Hiscock, 1981; Balbinot, 1986). Recently, we proposed a new method with which one can easily show both the location and the temperature of the event horizon of every evaporating black hole without calculating the energy-momentum tensor (Zhao and Dai, 1992). Here, we improve the method and make use of it to study the Vaidya–Bonner black hole (Bonner and Vaidya, 1970).

In the Vaidya–Bonner space-time

$$ds^2 = -\left(1 - \frac{2m(v)}{r} + \frac{Q^2(v)}{r^2}\right) dv^2 + 2\, dv\, dr + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \quad (1)$$

[1]Department of Physics, Beijing Normal University, Beijing 100875, China.
[2]Permanent address: Department of Physics, Fuzhou Teacher's College, Jiang Xi 344000, China.
[3]Permanent address: Department of Physics, Hese Education College, Shan Dong 274016, China.

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the Klein–Gordon equation

$$\frac{1}{(-g)^{1/2}}\left\{\left(\frac{\partial}{\partial x^{\mu}}-ieA_{\mu}\right)\left[(-g)^{1/2}g^{\mu\nu}\left(\frac{\partial}{\partial x^{\nu}}-ieA_{\nu}\right)\Phi\right]\right\}-\mu_0^2\Phi=0 \quad (2)$$

can be reduced to

$$\frac{1}{\sin\theta}\left(\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial S}{\partial\theta}\right)\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 S}{\partial\phi^2}+l(l+1)S=0 \quad (3)$$

$$\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)\frac{\partial^2\rho}{\partial r^2}+2\frac{\partial^2\rho}{\partial v\,\partial r}+\left(\frac{m}{r}-\frac{Q^2}{r^2}+ieQ\right)\frac{2}{r}\frac{\partial\rho}{\partial r}$$
$$-\left(\frac{2}{r^2}\left(\frac{m}{r}-\frac{Q^2}{r^2}\right)+\left(\mu_0^2+\frac{l(l+1)}{r^2}+\frac{ieQ}{r}\right)\right\}\rho=0 \quad (4)$$

where μ_0 and e are the mass and the electric charge of the Klein–Gordon particle, respectively, $A_{\mu}=(-Q/r,0,0,0)$, and $l=0,1,2,\ldots$. Here we have separated variables as

$$\Phi=\frac{1}{r}\rho(r,v)s(\theta,\phi) \quad (5)$$

Given the generalized tortoise coordinates

$$r_*=r+\frac{1}{2\varkappa}\ln[r-r_H(v)], \qquad v_*=v-v_0 \quad (6)$$

we can write the radial equation (4)

$$\left\{\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)\left(1+\frac{1}{2\varkappa(r-r_H)}\right)-\frac{2\dot{r}_H}{2\varkappa(r-r_H)}\right\}\frac{\partial^2\rho}{\partial r_*^2}+2\frac{\partial^2\rho}{\partial v_*\,\partial r_*}$$
$$+\left(\frac{2\varkappa(r-r_H)}{2\varkappa(r-r_H)+1}\right)\left\{\frac{2\dot{r}_H}{2\varkappa(r-r_H)^2}-\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)\frac{1}{2\varkappa(r-r_H)^2}\right.$$
$$\left.+2\left(\frac{m}{r^2}-\frac{Q^2}{r^3}+\frac{ieQ}{r}\right)\left(1+\frac{1}{2\varkappa(r-r_H)}\right)\right\}\frac{\partial\rho}{\partial r_*}-\left(1+\frac{1}{2\varkappa(r-r_H)}\right)^{-1}$$
$$\times\left\{\frac{2}{r^2}\left(\frac{m}{r}-\frac{Q^2}{r^2}\right)+\left(\mu_0^2+\frac{l(l+1)}{r^2}+\frac{ieQ}{r}\right)\right\}\rho=0 \quad (7)$$

where r_H is the location of the event horizon, and $\varkappa$ is an adjustable parameter. Both $\varkappa$ and v_0 are constant under the tortoise transformation (6), r_H can be determined by the null surface condition

$$g^{\mu\nu}\frac{\partial f}{\partial x^{\mu}}\frac{\partial f}{\partial x^{\nu}}=0 \quad (8)$$

In the Vaidya–Bonner space-time, equation (8) can be reduced to

$$r^2 - 2mr + Q^2 - 2r^2 \frac{dr}{dv} = 0 \tag{9}$$

r_H is just the solution of the equation. It means that r_H is shown as

$$r_H = \frac{m + [m^2 - Q^2(1 - 2\dot{r}_H)]^{1/2}}{1 - 2\dot{r}_H} \tag{10}$$

When r goes to $r_H(v_0)$ and v goes to v_0, equation (7) can be reduced to

$$B \frac{\partial^2 \rho}{\partial r_*^2} + 2 \frac{\partial^2 \rho}{\partial v_* \, \partial r_*} + 2 \frac{ieQ}{r_H} \frac{\partial \rho}{\partial r_*} = 0 \tag{11}$$

where

$$B \equiv \lim_{\substack{r \to r_H(v_0) \\ v \to v_0}} \frac{(r^2 - 2mr + Q^2)[2\varkappa(r - r_H) + 1] - 2r^2 \dot{r}_H}{2\varkappa r^2 (r - r_H)} \tag{12}$$

We select the adjustable parameter $\varkappa$ in equations (6) as

$$\varkappa = \frac{r_H - m - 2r_H \dot{r}_H}{2mr_H - Q^2} \tag{13}$$

Then we have $B = 1$; equation (11) can be written as

$$\frac{\partial^2 \rho}{\partial r_*^2} + 2 \frac{\partial^2 \rho}{2\partial v_* \, \partial r_*} + 2 \frac{ieQ}{r_H} \frac{\partial \rho}{\partial r_*} = 0 \tag{14}$$

The ingoing wave solution and the outgoing wave solution of the equations are, respectively,

$$\rho_{\text{in}} = e^{-i\omega v_*} \tag{15}$$

$$\rho_{\text{out}} = e^{-i\omega v_* + 2i(\omega - \omega_0) r_*} \tag{16}$$

where $\omega_0 = eQ/r_H$ is the electric potential at the horizon of the black hole. Equation (16) can be written as

$$\rho_{\text{out}} = e^{-i\omega v_* + 2i(\omega - \omega_0) r} (r - r_H)^{i(\omega - \omega_0)/\varkappa} \tag{17}$$

It is not analytical at the horizon r_H, but we can extend it by analytical continuation to the inside of the event horizon through the lower half complex r-plane,

$$(r - r_H) \to |r - r_H| \, e^{-i\pi} = (r_H - r) \, e^{-i\pi} \tag{18}$$

$$\rho_{\text{out}} \to \rho'_{\text{out}} = e^{-i\omega v_* + 2i(\omega - \omega_0) r} (r_H - r)^{i(\omega - \omega_0)/\varkappa} \, e^{\pi(\omega - \omega_0)/\varkappa} \tag{19}$$

The scattering probability of the outgoing wave at the horizon is

$$\left|\frac{\rho_{\text{out}}}{\rho'_{\text{out}}}\right|^2 = e^{-2\pi(\omega-\omega_0)/\varkappa} \tag{20}$$

Following Damour and Ruffini (1976) and Sannan (1988), it is easy to obtain the spectrum of radiation of the Klein–Gordon particles from the black hole:

$$N_\omega = (e^{(\omega-\omega_0)/k_B T} - 1)^{-1} \tag{21}$$

$$T = \frac{\varkappa}{2\pi k_B} = \frac{1}{2\pi k_B}\frac{r_H - m - 2r_H \dot{r}_H}{2mr_H - Q^2} \tag{22}$$

$$\omega_0 = eQ/r_H$$

where k_B is the Boltzmann's constant.

Now we deal with the Dirac equation near the event horizon of the Vaidya–Bonner black hole.

The spinor base form of the Dirac equation in curved space-time (Page, 1976), with signature -2, is

$$\begin{aligned}(\nabla_{a\dot{b}} + ieA_{a\dot{b}})p^a + i(\mu_0/\sqrt{2})\bar{Q}_{\dot{b}} &= 0\\ (\nabla_{a\dot{b}} - ieA_{a\dot{b}})Q^a + i(\mu_0/\sqrt{2})\bar{P}_{\dot{b}} &= 0\end{aligned} \tag{23}$$

where μ_0 and e are, respectively, the mass and the electric charge of the Dirac particle, P^a, Q^a, and $\nabla_{a\dot{b}}$ are the two-component spinors and the covariant spinor differentiation expressed with spinor base components, respectively. Equations (23) can be transformed into four coupled equations

$$\begin{aligned}(D + \varepsilon - \rho + ieA_\mu l^\mu)F_1 + (\bar{\delta} + \pi - \alpha + ieA_\mu \bar{m}^\mu)F_2 &= i(\mu_0/\sqrt{2})G_1\\ (\Delta + \mu - \gamma + ieA_\mu n^\mu)F_2 + (\delta + \beta - \tau + ieA_\mu m^\mu)F_1 &= i(\mu_0/\sqrt{2})G_2\\ (D + \bar{\varepsilon} - \bar{\rho} + ieA_\mu l^\mu)G_2 - (\delta + \bar{\pi} - \bar{\alpha} + ieA_\mu m^\mu)G_1 &= i(\mu_0/\sqrt{2})F_2\\ (\Delta + \bar{\mu} - \bar{\gamma} + ieA_\mu n^\mu)G_1 - (\bar{\delta} + \bar{\beta} - \bar{\tau} + ieA_\mu \bar{m}^\mu)G_2 &= i(\mu_0/\sqrt{2})F_1\end{aligned} \tag{24}$$

where

$$\begin{aligned}F_1 = P^0, \qquad F_2 = P^1, \qquad G_1 &= \bar{Q}^{\dot{1}}, \qquad G_2 = -\bar{Q}^{\dot{0}}\\ D = \partial_{0\dot{0}} = l^\mu \partial_\mu, \qquad \Delta &= \partial_{1\dot{1}} = n^\mu \partial_\mu\\ \delta = \partial_{0\dot{1}} = m^\mu \partial_\mu, \qquad \bar{\delta} &= \bar{m}^\mu \partial_\mu\end{aligned} \tag{25}$$

μ, γ, β, τ, ε, ρ, π, and α are the special designations of the spin coefficients defined by Newman and Penrose (1962). l^μ, n^μ, m^μ, and $\bar{m}^\mu$ are the null tetrad vectors; they satisfy

$$
\begin{aligned}
l_\mu l^\mu &= n_\mu n^\mu = m_\mu m^\mu = \bar{m}_\mu \bar{m}^\mu = 0 \\
l_\mu m^\mu &= l_\mu \bar{m}^\mu = n_\mu m^\mu = n_\mu \bar{m}^\mu = 0 \\
l_\mu n^\mu &= -m_\mu \bar{m}^\mu = 1 \\
g_{\mu\nu} &= l_\mu n_\nu + n_\mu l_\nu - m_\mu \bar{m}_\nu - \bar{m}_\mu m_\nu
\end{aligned} \tag{26}
$$

In the Vaidya–Bonner space-time, they are

$$
\begin{aligned}
l_\mu &= (1/2\Delta)^{1/2}(\Delta/r, 0, 0, 0) \\
n_\mu &= (1/2\Delta)^{1/2}(\Delta/r, -2r, 0, 0) \\
m_\mu &= (r/\sqrt{2})(0, 0, 1, i \sin\theta) \\
\bar{m}_\mu &= (r/\sqrt{2})(0, 0, 1, -i \sin\theta)
\end{aligned} \tag{27}
$$

and

$$
\begin{aligned}
l^\mu &= (1/2\Delta)^{1/2}(0, \Delta/r, 0, 0) \\
n^\mu &= (1/2\Delta)^{1/2}(2r, \Delta/r, 0, 0) \\
m_\mu &= (1/\sqrt{2}\, r)(0, 0, -1, -i/\sin\theta) \\
\bar{m}^\mu &= (1/\sqrt{2}\, r)(0, 0, -1, i/\sin\theta)
\end{aligned} \tag{28}
$$

We calculate the spin coefficients and get

$$
\begin{aligned}
\pi &= \tau = 0 \\
\varepsilon &= -\frac{1}{2}\left(\frac{1}{2\Delta}\right)^{1/2} \frac{mr - Q^2}{r^2} \\
\rho &= \mu = \left(\frac{1}{2\Delta}\right)^{1/2} \frac{\Delta}{r^2} \\
\alpha &= -\beta = (1/2\sqrt{2}\, r) \operatorname{cgt} \theta \\
\gamma &= \left(\frac{1}{2\Delta}\right)^{1/2} \left[\frac{r}{\Delta}(Q\dot{Q} - \dot{m}r) - \frac{mr - Q^2}{r^2}\right]
\end{aligned} \tag{29}
$$

The differentiation operators are

$$
\begin{aligned}
D &= -\left(\frac{1}{2\Delta}\right)^{1/2} \frac{\Delta}{r} \frac{\partial}{\partial r} \\
\Delta &= \left(\frac{1}{2\Delta}\right)^{1/2} \left(2r \frac{\partial}{\partial v} + \frac{\Delta}{r} \frac{\partial}{\partial r}\right) \\
\delta &= -\frac{1}{\sqrt{2}\, r} \left(\frac{\partial}{\partial \theta} + \frac{i}{\sin\theta} \frac{\partial}{\partial \phi}\right) \\
\bar{\delta} &= -\frac{1}{\sqrt{2}\, r} \left(\frac{\partial}{\partial \theta} - \frac{i}{\sin\theta} \frac{\partial}{\partial \phi}\right)
\end{aligned} \tag{30}
$$

Substituting them into equation (24) and separating variables as

$$\begin{aligned} G_1 &= e^{iv\phi}R_+(v, r)S_-(\theta) \\ G_2 &= e^{iv\phi}R_-(v, r)S_+(\theta) \\ F_1 &= e^{iv\phi}R_-(v, r)S_-(\theta) \\ F_1 &= e^{iv\phi}R_+(v, r)S_+(\theta) \end{aligned} \tag{31}$$

we have

$$\begin{aligned} &\mathscr{D}_0\mathscr{D}_1R_+ + \frac{\lambda - i\mu_0 r}{\sqrt{\Delta}}\left(\mathscr{D}_0\frac{\sqrt{\Delta}}{\lambda - i\mu_0 r}\right)\mathscr{D}_1R_+ - \frac{1}{\Delta}(\lambda^2 + \mu_0^2 r^2)R_+ = 0 \\ &\mathscr{D}_1\mathscr{D}_0R_- + \frac{\lambda + i\mu_0 r}{\sqrt{\Delta}}\left(\mathscr{D}_1\frac{\sqrt{\Delta}}{\lambda + i\mu_0 r}\right)\mathscr{D}_0R_- - \frac{1}{\Delta}(\lambda^2 + \mu_0^2 r^2)R_- = 0 \\ &\mathscr{L}_-\mathscr{L}_+S_+ + \lambda^2 S_+ = 0 \\ &\mathscr{L}_+\mathscr{L}_-S_- + \lambda^2 S_- = 0 \end{aligned} \tag{32}$$

where

$$\mathscr{D}_0 = \frac{\partial}{\partial r} + \frac{1}{\Delta}\frac{2r^2 - 3mr + Q^2}{2r}$$

$$\mathscr{D}_1 = \frac{2r^2}{\Delta}\frac{\partial}{\partial v} + \frac{\partial}{\partial r} + \frac{1}{\Delta}\frac{2r^2 - 3mr + Q^2}{2r} - \frac{r^2}{\Delta^2}(Q\dot{Q} - \dot{m}r) + \frac{i2eQr}{\Delta}$$

$$\mathscr{L}_\pm = \frac{\partial}{\partial\theta} \pm \frac{v}{\sin\theta} + \frac{1}{2}\operatorname{ctg}\theta$$

Here λ is a constant introduced by the separation of variables.

Now, let us study the equation for R_+ in (32). It can be rewritten as

$$\begin{aligned} &\frac{\partial^2 R_+}{\partial r^2} + \frac{2r^2}{\Delta}\frac{\partial^2 R_+}{\partial v\,\partial r} + \left\{\frac{2r^2}{\Delta}\left(-\frac{\mu_0^2 r - i\mu_0\lambda}{\lambda^2 + \mu_0^2 r^2} + \frac{r-m}{\Delta} + \frac{1}{\Delta}\frac{2r^2 - 3mr + Q^2}{2r}\right)\right. \\ &\left. + \frac{r}{\Delta^2}(2r^2 - 7mr + 5Q^2)\right\}\frac{\partial R_+}{\partial r} + \left(-\frac{\mu_0^2 r - i\mu_0\lambda}{\lambda^2 + \mu_0^2 r^2} + \frac{r-m}{\Delta}\right. \\ &\left. + \frac{3}{\Delta}\frac{2r^2 - 3mr + Q^2}{2r} - \frac{r^2}{\Delta^2}(Q\dot{Q} - \dot{m}r) + \frac{i2eQr}{\Delta}\right)\frac{\partial R_+}{\partial r} \\ &+ \left\{\left(-\frac{r^2}{\Delta^2}(Q\dot{Q} - \dot{m}r) + \frac{i2eQr}{\Delta}\right)\left(-\frac{\mu_0^2 r - i\mu_0\lambda}{\lambda^2 + \mu_0^2 r^2} + \frac{r-m}{\Delta}\right.\right. \\ &\left. + \frac{1}{\Delta}\frac{2r^2 - 3mr + Q^2}{2r}\right) - \frac{r-m}{\Delta^2}\frac{2r^2 - 3mr + Q^2}{r} + \frac{1}{\Delta}\frac{2r^2 - Q^2}{2r^2} \end{aligned}$$

$$
\begin{aligned}
&+\frac{4r^2}{\Delta^3}(r-m)(Q\dot{Q}-\dot{m}r)-\frac{r}{\Delta^2}(2Q\dot{Q}-3\dot{m}r)+i2eQ\frac{Q^2-r^2}{\Delta^2}\\
&+\frac{1}{\Delta}\frac{2r^2-3mr+Q^2}{2r}\left(\frac{1}{\Delta}\frac{2r^2-3mr+Q^2}{2r}-\frac{r^2}{\Delta^2}(Q\dot{Q}-\dot{m}r)+\frac{i2eQr}{\Delta}\right)\\
&\left.-\frac{1}{\Delta}(\lambda^2+\mu^2 r)\right\}R_+=0
\end{aligned}\tag{33}
$$

By means of the generalized tortoise coordinates (6), we can write this equation as

$$
\begin{aligned}
&\frac{\Delta[2\varkappa(r-r_H)+1]-2r^2\dot{r}_H}{2\varkappa r^2(r-r_H)}\frac{\partial^2 R_+}{\partial r_*^2}+\frac{\partial^2 R_+}{\partial v_*\,\partial r_*}+\frac{2\varkappa(r-r_H)}{2\varkappa(r-r_H)+1}\\
&\times\left[2\left(-\frac{\mu_0^2 r-i\mu_0\lambda}{\lambda^2+\mu_0^2 r^2}+\frac{r-m}{\Delta}+\frac{1}{\Delta}\frac{2r^2-3mr+Q^2}{2r}\right)\right.\\
&\left.+\frac{1}{r\Delta}(2r^2-7mr+5Q^2)\right]\frac{\partial R_+}{\partial v_*}+\left\{\frac{2r^2\dot{r}_H-\Delta}{r^2(r-r_H)[2\varkappa(r-r_H)+1]}\right.\\
&-\frac{\dot{r}_H}{2\varkappa(r-r_H)+1}\left[2\left(-\frac{\mu_0^2 r-i\mu_0\lambda}{\lambda^2+\mu_0^2 r^2}+\frac{r-m}{\Delta}+\frac{1}{\Delta}\frac{2r^2-3mr+Q^2}{2r}\right)\right.\\
&\left.+\frac{1}{r\Delta}(2r^2-7mr+5Q^2)\right]-\frac{\Delta}{r^2}\frac{\mu_0^2 r-i\mu_0\lambda}{\lambda^2+\mu_0^2 r^2}+\frac{r-m}{r^2}+\frac{3}{2}\\
&\left.\times\frac{2r^2-3mr+Q^2}{r^3}-\frac{Q\dot{Q}-\dot{m}r}{\Delta}+i\frac{2eQ}{r}\right\}\frac{\partial R_+}{\partial r_*}+\frac{2\varkappa(r-r_H)}{2\varkappa(r-r_H)+1}\\
&\times\left\{\left(-\frac{Q\dot{Q}-\dot{m}r}{\Delta}+i\frac{2eQ}{r}\right)\left(-\frac{\mu_0^2 r-i\mu_0\lambda}{\lambda^2+\mu_0^2 r^2}+\frac{r-m}{\Delta}+\frac{1}{\Delta}\frac{2r^2-3mr+Q^2}{2r}\right)\right.\\
&-\frac{r-m}{\Delta}\frac{2r^2-3mr+Q^2}{r^3}+\frac{2r^2-Q^2}{2r^4}+\frac{4}{\Delta^4}(r-m)(Q\dot{Q}-\dot{m}r)\\
&-\frac{1}{r\Delta}(2Q\dot{Q}-3\dot{m}r)+i2eQ\frac{Q^2-r^2}{r^2\Delta}+\frac{2r^2-3mr+Q^2}{2r^3}\\
&\left.\times\left[\frac{1}{\Delta}\frac{2r^2-3mr+Q^2}{2r}-\frac{r^2}{\Delta^2}(Q\dot{Q}-\dot{m}r)+\frac{i2eQr}{\Delta}\right]-\frac{1}{\Delta}(\lambda^2+\mu_0^2 r)\right\}R_+=0
\end{aligned}\tag{34}
$$

When r approaches $r_H(v_0)$ and v approaches v_0, it can be reduced to

$$
\frac{\partial^2 R_+}{\partial r_*^2}+2\frac{\partial^2 R_+}{\partial v_*\,\partial r_*}+(\alpha_0+i2\omega_0)\frac{\partial R_+}{\partial r_*}=0 \tag{35}
$$

where

$$\alpha_0 = \frac{2r_H^2 - 3mr_H + Q^2}{2r_H^3} - \frac{Q\dot{Q} - \dot{m}r_H}{2r_H^2\dot{r}_H}$$

$$\omega_0 = +\frac{eQ}{r_H} \tag{36}$$

The ingoing wave solution and the outgoing wave solution, are respectively,

$$R_+^{\text{in}} \sim e^{-i\omega v_*}$$

$$R_+^{\text{out}} \sim e^{-i\omega v_*}\, r^{2i(\omega - \omega_0)r_*}\, e^{-\alpha_0 r_*} \qquad (r > r_H) \tag{38}$$

The outgoing wave (38) is not analytic at the event horizon $r = r_H$. But we can analytically continue it from the outside of the black hole into the inside of the black hole, so we have

$$\tilde{R}_+^{\text{out}} \sim e^{-i\omega v_*}\, e^{2i(\omega - \omega_0)r_*}\, e^{-\alpha_0 r_*}\, e^{\pi(\omega - \omega_0)/\varkappa}\, e^{i\pi\alpha_0/2\varkappa} \qquad (r < r_H) \tag{39}$$

The relative scattering probability produced by the event horizon is

$$\left|\frac{R_+^{\text{out}}}{\tilde{R}_+^{\text{out}}}\right|^2 = e^{-2\pi(\omega - \omega_0)/\varkappa} \tag{40}$$

Similarly, we get the spectrum of Hawking radiation of the Dirac particles from the Vaidya–Bonner black hole,

$$N_\omega = (e^{(\omega - \omega_0)/k_B T} + 1)^{-1} \tag{41}$$

where T is given by equation (22).

Equations (10) and (22) give the location and the temperature of the event horizon of the Vaidya–Bonner black hole. Equations (21) and (41) show the Hawking thermal spectra of the Klein–Gordon particles and the Dirac particles in the Vaidya–Bonner space-time.

When $m \gg Q$ and

$$\dot{m} \sim Q\dot{Q}/m \sim Q^2/m^2 \tag{42}$$

is very small, we have

$$\dot{r}_H = \dot{r}_{AH} = 2\dot{m} - Q\dot{Q}/m \tag{43}$$

where r_{AH} is the apparent horizon of the black hole. Substituting equation (43) into equations (10) and (22), we get

$$r_H = 2m(1 + 4\dot{m} - 2Q\dot{Q}/m - Q^2/4m^2) \tag{44}$$

$$T = (1/8\pi m)(1 - 4\dot{m} + 2Q\dot{Q}/m) \tag{45}$$

When $Q = 0$, equations (10) and (22) become the result of the Vaidya black hole. When $\dot{m} = \dot{Q} = 0$, they become that of the static Reissner–Nordström black hole.

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